D Supplementary Material for: "The Spillover Effects of Top Income Inequality"

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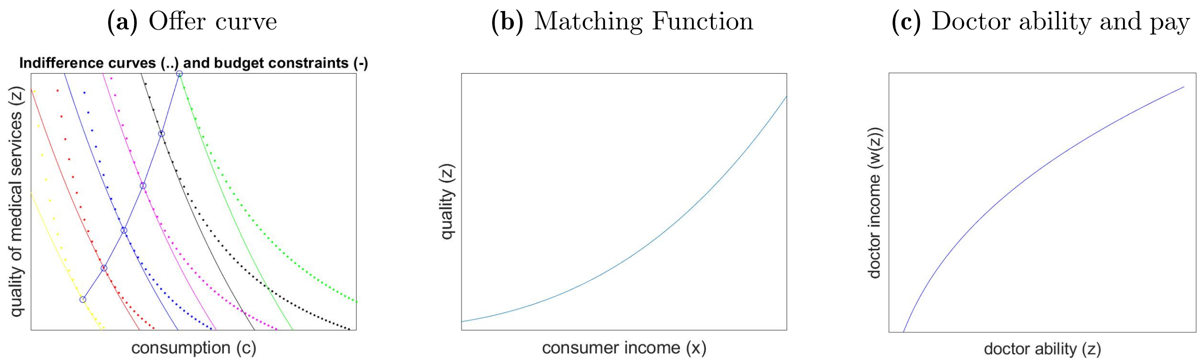
D.1 Graphical representation of the model

Figure D.1: Equilibrium in the baseline model when $\alpha_z < \alpha_x$



Notes: This figure illustrates the matching mechanism in the model when $\alpha_z < \alpha_x$. Panel (a) shows the budget sets and indifference curves for six different consumers, along with the matching function that this equilibrium generates. The horizontal axis shows consumption c of the homogeneous good, and the vertical axis shows the quality of physician z that the consumer obtains. The dotted curves represent the indifference curves, and the solid curves the budget constraints. The budget constraints are curved because there is not a constant price per unit of quality; in this example, additional units of quality have decreasing cost. So, for any given budget constraint, the constraint flattens as we move to the left. Because there is much more skill dispersion among doctors than among consumer (the ability distribution of doctors has a fatter tail), the higher-income consumers have a increasingly abundant medical services. Consequently, the quality of physician is convex in consumer income (Panel (b)). Panel (c) shows the income of physicians.

We illustrate equilibrium of the baseline model in Figure D.1. Panel D.1a shows the budget sets and indifference curves for six consumers with different income levels. For this illustration we choose $\alpha_z < \alpha_x$; that is, skill inequality among physicians is higher than among widget makers. For each consumer, the horizontal axis shows consumption c of the homogeneous good, and the vertical axis shows the quality of the physician z that the consumer obtains. The dotted curves represent the indifference curves, and the solid curves the budget constraints. With Cobb-Douglas utility, the higher indifference curves are proportionally scaled versions of lower indifference curves—the slopes are constant on any ray out from the origin. But the budget constraints behave differently from those with constant relative prices: they are curved because there is not a constant price per unit of quality. In this example, additional units of quality have decreasing cost, due to the relative abundance of skill for physicians in the top. So, for any given budget constraint, the budget line becomes steeper as we move to the left. Income differences lead to parallel shifts left or right in the budget constraint. As a result, the slopes at which the indifference curves are tangent to the budget constraint change for different budget constraints. Consequently, the curve that traces out optimal bundles for increasing income is not a straight line from the origin, as would be the case for Cobb-Douglas utility and constant prices, but instead a convex function. Panel D.1b shows the matching function: Since $\alpha_z < \alpha_x$ it must match widget makers with increasingly high-skill physicians, implying a convex matching function. Finally, Panel D.1c shows the income of physicians with a given ability. With divisible services, Panel D.1c would be a straight line.

D.2 Disentangling supply side and demand side effects

As described in the text of Section 3.2.1, in the model with occupational mobility, doctors and widget makers interact through both demand and outside option effects. To disentangle these two effects, we now build a model where doctors have an outside option positively correlated with their ability but where patients are a separate group. Formally, there are two types of agents: a mass 1 of consumers, with income x distributed with the Pareto distribution $P(X > x) = (x_{\min}/x)^{\alpha_x}$ and a mass M of potential doctors. Consumers consume the homogeneous good and health care services according to the utility function (1). Potential doctors only consume the homogeneous good. As in section 3.2.1, they are ranked in descending order of ability and we denote i their rank. Agent i can choose between being a doctor providing health services of quality $z(i)$ and earning $w(z(i))$ or working in the homogeneous good sector earning $y(i)$. y and z are distributed according to the counter-cumulative distributions:

$$\overline{G}_y(y(i)) = \overline{G}_z(z(i)) = i \text{ with } \overline{G}_y = (y_{\min}/y)^{\alpha_y} \text{ and } \overline{G}_z = (z_{\min}/z)^{\alpha_z}.$$

Further $\lambda M > 1$ so that all consumers can get health services.

Assume that the equilibrium is such that for individuals of a sufficiently high level of ability, some choose to be doctors and others to work in the homogeneous good sector. Then, for i low enough, agents must be indifferent between becoming a doctor or working in the homogeneous good sector, so that $w(z(i)) = y(i)$. Hence, the wage function obeys:

$$w(z) = y_{\min}(z/z_{\min})^{\alpha_z/\alpha_y}. \tag{43}$$

Market clearing for health care services above z implies:

$$\left(\frac{x_{\min}}{m(z)}\right)^{\alpha_x} = \lambda M \int_z^{\infty} \mu(\zeta)\, g_z(\zeta)\, d\zeta, \tag{44}$$

where $\mu(\zeta)$ denotes the share of potential doctors who choose to be doctors. Plugging this expression in the first order condition (2) together with (43), we obtain:

$$\int_{z}^{\infty} \mu(\zeta)\, g_z(\zeta)\, d\zeta = \frac{1}{\lambda M} \left(\frac{\frac{\beta}{1-\beta} \lambda x_{\min}}{\left(\frac{\alpha_z}{\alpha_y} + \frac{\beta}{1-\beta} \right) y_{\min}} \right)^{\alpha_x} \left(\frac{z}{z_{\min}} \right)^{-\alpha_x \frac{\alpha_z}{\alpha_y}}. \tag{45}$$

Taking the derivative with respect to z, we get:

$$\mu(z) = \frac{\alpha_x}{\alpha_y} \frac{1}{\lambda M} \left(\frac{\frac{\beta}{1-\beta} \lambda x_{\min}}{\left(\frac{\alpha_z}{\alpha_y} + \frac{\beta}{1-\beta} \right) y_{\min}} \right)^{\alpha_x} \left(\frac{z}{z_{\min}} \right)^{\alpha_z \left(1 - \frac{\alpha_x}{\alpha_y}\right)}. \tag{46}$$

Since $\mu(z) \in (0,1)$, this case is only possible if $\alpha_y \leq \alpha_x$, that is consumers' income distribution has a fatter tail than the outside option for potential doctors (and, if $\alpha_y = \alpha_x$, $\frac{\alpha_x}{\alpha_y} \frac{1}{\lambda M} \left(\frac{\alpha_y \beta \lambda x_{\min}}{(\alpha_z(1-\beta) + \beta \alpha_y) y_{\min}} \right)^{\alpha_x} \leq 1$). We then obtain that doctors' income distribution obeys (for w high enough):

$$\Pr(W > w) = \int_{z_{\min}\left(\frac{w}{y_{\min}}\right)^{\frac{\alpha_y}{\alpha_z}}}^{\infty} \mu(\zeta) \left(\frac{z_{\min}}{\zeta} \right)^{\alpha_z} \frac{d\zeta}{\zeta} = \frac{1}{\lambda M \alpha_z} \left(\frac{\alpha_y \beta \lambda x_{\min}}{\alpha_z (1-\beta) + \beta \alpha_y} \right)^{\alpha_x} w^{-\alpha_x}.$$

Therefore doctors' income is distributed like the patients' income and not according to doctors' outside option.

With $\alpha_y > \alpha_x$ or $\alpha_y = \alpha_x$ together with $\frac{\alpha_x}{\alpha_y} \frac{1}{\lambda M} \left(\frac{\alpha_y \beta \lambda x_{\min}}{(\alpha_z(1-\beta) + \beta \alpha_y) y_{\min}} \right)^{\alpha_x} > 1$, then above a certain threshold, all potential doctors will choose to be doctors, so that the model behaves like that of section 3.1, and the outside option is "mute".

Therefore, in all cases, income is Pareto distributed at the top with shape parameter α_x. Changes in α_y have no impact on doctors' top income inequality. This result, however, relies on the Cobb-Douglas assumption and does not generalize to a CES case.